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PARAMETRIC METHOD OF SOLVING A NONLINEAR HEAT-CONDUCTION PROBLEM FOR A SEMIINFINITE BODY

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A new method is proposed for solving heat-conduction problems with nonlinear boundary conditions.

A very essential shortcoming of the well-known integral methods for solving nonlinear heat-conduction problems [1] is the a priori choice of the family of temperature profiles or of heat-flux density. The degree of approximation of the adopted distribution of the sought values to the true one and thus the error of the method depends one one's intuition; as a rule, they are only satisfactory in a finite range of the values of the parameter.

Up to the mid-1960's a similar situation could be observed as regards the related problem of evaluating the laminar boundary layer when the multiparameter method developed by Loitsyanskii [2] was published, showing the way for obtaining the families of the velocity profiles in the boundary-layer section in a rational manner. It was based on solving the boundary-layer differential equation in new dimensionless parameters (the similarity parameters), thus ensuring good accuracy of the obtained results when analyzing specific problems.

In this article an attempt is made to generalize the concepts of the Loitsyanskii method to the nonlinear problems of heat conduction.

We now consider a heat-conduction problem for a semiinfinite body with constant thermal characteristics, which can be formulated as follows:

$$\frac{\partial T(x,\tau)}{\partial \tau} = a\frac{\partial^2 T(x,\tau)}{\partial x^2}; \tag{1}$$

$$\begin{cases} -\lambda\dfrac{\partial T}{\partial x} = Q(T_p\ \ T_s,\ \tau) \ \text{for}\ \ x=0, \\ \dfrac{\partial T}{\partial x} = 0,\ T = T_\infty \ \ \text{as}\ \ x\to\infty; \end{cases} \tag{2}$$

$$T = T_0(x)\ \text{for}\ \ \tau = 0. \tag{3}$$

In this form, the problem (1)-(3) is referred to according to the classification of [1] as a nonlinear problem of the second kind, where the nonlinearity appears only in the boundary conditions (2).

Instead of the variable $T(x, \tau)$, the variable $q(x, \tau)$ is introduced by means of the relation

$$q(x,\tau) = -\lambda\frac{\partial T(x,\tau)}{\partial x}, \tag{4}$$

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and then the dimensionless variable

$$\varphi(x, \tau) = q(x, \tau)/Q(T_p, T_s, \tau). \tag{5}$$

Then the problem (1)-(3) can be rewritten as

$$\frac{\varphi}{Q}\frac{dQ}{d\tau} + \frac{\partial\varphi}{\partial\tau} = a\frac{\partial^2\varphi}{\partial x^2}; \tag{6}$$

$$\begin{cases}\varphi = 1 \text{ for } x = 0,\\ \varphi = 0 \text{ as } x \to \infty;\end{cases} \tag{7}$$

$$\varphi = -\frac{\lambda}{Q}\frac{\partial T_0}{\partial x} = \varphi_0(x) \text{ for } \tau = 0. \tag{8}$$

The thickness of the "filling-up"

$$\delta^*(\tau) = \int_0^\infty \varphi(x, \tau)\,dx \tag{9}$$

is adopted as a measure of performance, characterizing at every time instant the size of the region of temperature variation; the following dimensionless coordinate is introduced:

$$\eta(x, \tau) = x/\delta^*(\tau). \tag{10}$$

By making use of the relations

$$\frac{\partial^2}{\partial x^2} = \frac{1}{\delta^{*2}}\frac{\partial^2}{\partial\eta^2}, \quad \frac{\partial}{\partial\tau} = \left(\frac{\partial}{\partial\tau}\right) - \frac{\eta}{\delta^*}\frac{d\delta^*}{d\tau}\frac{\partial}{\partial\eta},$$

where $\partial/\partial\tau$ denotes the explicit partial differentiation only, and employing (6), one obtains

$$\frac{\partial^2\varphi}{\partial\eta^2} + \frac{1}{2}\eta F\frac{\partial\varphi}{\partial\eta} - f_1\varphi = z\frac{\partial\varphi}{\partial\tau}. \tag{11}$$

The following notation has been introduced:

$$F(\tau) = dz/d\tau, \tag{12}$$

$$z(\tau) = \delta^{*2}/a, \tag{13}$$

$$f_1(\tau) = \frac{z}{Q}\frac{dQ}{d\tau}. \tag{14}$$

The quantity $f_1(\tau)$ may be regarded as the first term of the parameter series which can be evaluated by using the general formula,

$$f_k(\tau) = \frac{z^k}{Q}\frac{d^kQ}{d\tau^k}. \tag{15}$$

By differentiating (15) it can easily be seen that there is the following recurrence relation between the terms of the series:

$$z\,df_k/d\tau = \omega_k, \tag{16}$$

where

$$\omega_k = kFf_k + f_{k+1} - f_1 f_k. \tag{17}$$

The equation for heat balance is now obtained. To this end, Eq. (6) is integrated with respect to x from 0 to ∞. By introducing the dimensionless coordinate η, one obtains

$$\delta^*\frac{d\delta^*}{d\tau} + \frac{\delta^{*2}}{Q}\frac{dQ}{d\tau} = -a\left.\frac{\partial\varphi}{\partial\eta}\right|_{\eta=0}. \tag{18}$$

Hence, by using (12)-(14) an expression for the quantity F is obtained:

$$F = -2\left(f_1 + \frac{\partial\varphi}{\partial\eta}\bigg|_{\eta=0}\right). \tag{19}$$

Thus, as a result of the transformations carried out the heat-conduction equation (1) has been reduced to its dimensionless form (11), and the heat-balance equation (18) obtained from the latter has assumed the form (12) with (19) taken into account.

Following [2], the dimensionless multiparameter representation of the heat fluxes $q/Q = \varphi(\eta, f_1, f_2, \ldots)$ is considered; the quantities f_k can be represented as parameters of generalized similarity.

It will be shown that the dimensionless variable φ which is a function of an infinite number of independent variables η, f_1, $f_2, \ldots$, is universal in the sense that it remains the same whatever the distribution (with respect to time) of the density of the external heat flux $Q(\tau)$.

It will be assumed in our further considerations that the function $\varphi(\eta, f_1, f_2, \ldots)$ of an infinite number of arguments exists and is continuous, its derivatives with respect to all variables also being continuous.

We proceed in Eq. (11) to other independent variables f_1, $f_2, \ldots$ by means of the formula

$$z\frac{\partial}{\partial\tau} = \sum_{k=1}^{\infty}\omega_k\frac{\partial}{\partial f_k},$$

which can be obtained from (16). Thus, one obtains the sought universal equation

$$\frac{\partial^2\varphi}{\partial\eta^2} + \frac{1}{2}\eta F\frac{\partial\varphi}{\partial\eta} - f_1\varphi = \sum_{k=1}^{\infty}\omega_k\frac{\partial\varphi}{\partial f_k}, \tag{20}$$

which should be analyzed with the boundary conditions

$$\varphi = 1 \text{ for } \eta = 0;\ \varphi = 0 \text{ as } \eta\to\infty;\ \varphi = \varphi_0(\eta) \text{ for } f_1 = f_2 = \ldots = 0, \tag{21}$$

obtained from (7) and (8) by proceeding to other variables, which are also universal in the same sense.

The last of the conditions (21) is found by using Eq. (20) itself. By setting $f_1 = f_2 = \ldots = 0$ in (20), one obtains

$$\frac{d^2\varphi_0}{d\eta^2} - \frac{d\varphi_0}{d\eta}\eta\frac{d\varphi_0}{d\eta}\bigg|_{\eta=0} = 0.$$

Hence, by using the first two conditions of (21), one finds

$$\varphi_0 = 1 - \frac{2}{\sqrt{\pi}}\int_0^u \exp(-u^2)\,du = \operatorname{erfc} u, \tag{22}$$

where $u = \eta/\sqrt{\pi}$.

The obtained result is identical to the solution of the linear heat-conduction problem with the boundary conditions of the second kind (Q = const, $q_0 = 0$) [3]. This can easily be established by integrating (12) with the initial conditions $z_0 = 0$ for $\tau = 0$, which leads to $z = 4\tau/\pi$; the latter gives, with the aid of (10) and (13), an explicit form for the variable u:

$$u = x/2\sqrt{a\tau}.$$

The nonlinear problem (20), (21) can only be solved if there is a finite number of variables f_1, $f_2 \ldots$. A single-parameter approximation is now considered; this corresponds to the assumption $f_1 \neq 0$, $f_2 = f_3 = \ldots = 0$. In this case the problem (20), (21) becomes

$$\frac{\partial^2\varphi}{\partial\eta^2} + \frac{1}{2}\eta F\frac{\partial\varphi}{\partial\eta} - f_1\varphi = \omega_1\frac{\partial\varphi}{\partial f_1}, \tag{23}$$

where, in agreement with (17), one has $\omega_1 = Ff_1 - f_1^2$;

$$\varphi = 1 \text{ for } \eta = 0;\ \varphi = 0 \text{ as } \eta\to\infty;\ \varphi = \operatorname{erfc}(\eta/\sqrt{\pi}) \text{ for } f_1 = 0. \tag{24}$$

TABLE 1. Values of the Functions $F^{(1)}(f_1)$ and $\Phi^{(1)}(\eta, f_1)$

$F(f_1)$	f_1 \ η	$\Phi(\eta, f_1)$ 0	0,2	0,4	0,6	0,8	1,0	1,2	1,4	1,6	1,8	2,0
1,2732	0	1,0000	0,8127	0,6505	0,5125	0,3971	0,3024	0,2263	0,1663	0,1199	0,0848	0,0588
1,5523	—0,2	1,0000	0,8117	0,6474	0,5070	0,3898	0,2940	0,2176	0,1581	0 1127	0,0790	0,0546
1,8324	—0,4	1,0000	0,8107	0,6443	0,5017	0,3827	0,2860	0,2093	0,1502	0,1057	0,0732	0,0501
2,1156	—0,6	1,0000	0,8098	0,6414	0,4968	0,3762	0,2785	0,2017	0,1430	0,0994	0,0680	0,0460
2,4022	—0,8	1,0000	0,8089	0,6386	0,4920	0,3700	0,2716	0,1947	0,1365	0,0938	0,0635	0,0425
2,6918	—1,0	1,0000	0,8080	0,6359	0,4875	0,3641	0,2651	0,1882	0,1306	0,0888	0,0595	0,0396
2,9340	—1,2	1,0000	0,8072	0,6333	0,4832	0,3586	0,2590	0,1823	0,1252	0,0843	0,0559	0,0370
3,2757	—1,4	1,0000	0,8063	0,6308	0,4791	0,3534	0,2533	0,1767	0,1200	0,0802	0,0528	0,0347
3,5754	—1,6	1,0000	0,8055	0,6284	0,4751	0,3484	0,2479	0,1716	0,1158	0,0765	0,0500	0,0327
3,8741	—1,8	1,0000	0,8047	0,6261	0,4713	0,3436	0,2428	0,1667	0,1115	0,0731	0,0475	0,0310
4,1745	—2,0	1,0000	0,8040	0,6238	0,4676	0,3390	0,2380	0,1622	0,1076	0,0700	0,0452	0,0294
4,4766	—2,2	1,0000	0,8032	0,6216	0,4640	0,3346	0,2334	0,1578	0,1040	0,0672	0,0431	0,0280

This approximation becomes exact if the density distribution of the external heat flux $Q(\tau)$ is linear in τ.

Equation (23) with the boundary conditions (24) was solved numerically by using the scheme of the sweep method [4]. The integration was carried out with the step $\Delta\eta = -\Delta f_1 = 0.1$ until $\eta = 7$ and $f_1 = -2.3$. By virtue of the universality of (23) it can once and for ever be integrated and its result tabulated. Having found the solutions $\varphi^{(1)}(\eta, f_1)$, one can also find and tabulate the quantities

$$F^{(1)}(f_1) = -2\left(f_1 + \left.\frac{\partial\varphi^{(1)}}{\partial\eta}\right|_{\eta=0}\right), \quad (25)$$

$$\Phi^{(1)}(\eta, f_1) = 1 - \int_0^\eta \varphi^{(1)}(\eta, f_1)\,d\eta. \quad (26)$$

The obtained results are given in Table 1. [From now on the superscript (1) is omitted.]

The function $\Phi(\eta, f_1)$ represents the dimensionless temperature profile in the body. The transition to dimensional quantities uses the formula

$$T = T_\infty + \frac{Q\sqrt{az}}{\lambda}\,\Phi(\eta, f_1), \quad (27)$$

which follows from the relation (4), the latter having been integrated with respect to x from x to ∞ and having proceeded to the new variables. This implies, in particular, that

$$T_\text{p} = T_\infty + \frac{Q\sqrt{az}}{\lambda}, \quad (28)$$

$$\frac{dT_\text{p}}{d\tau} = \frac{Q}{\lambda}\sqrt{\frac{a}{z}}\left(f_1 + \frac{F}{2}\right). \quad (29)$$

To find the final form of the solution for the problem (1)-(3) with a given specific function $Q(T_\text{p}, T_\text{s}, \tau)$ it is necessary to find, first of all, the dependence of the parameter f_1 and the quantity z on time. This can be achieved by solving Eq. (12), which in a single-parameter approximation becomes

$$dz/d\tau = F(f_1). \quad (30)$$

The relation between f_1 and z is determined by the formula (14), in which by virtue of (2) one has

$$\frac{dQ}{d\tau} = \frac{\partial Q}{\partial\tau} + P(T_\text{p}, T_\text{s}, \tau)\frac{dT_\text{p}}{d\tau} + R(T_\text{p}, T_\text{s}, \tau)\frac{dT_\text{s}}{d\tau}. \quad (31)$$

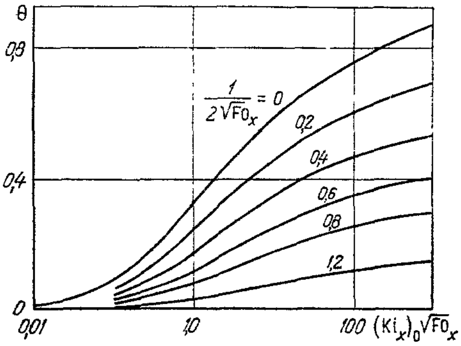

Fig. 1. The function $\Theta[(\mathrm{Ki}_x)_0\sqrt{\mathrm{Fo}_x}]$ for radiative heat transfer for $T_s = 0$ (m = 4).

In the above

$$P(T_p, T_s, \tau) = \frac{\partial Q}{\partial T_p}, \; R(T_p, T_s, \tau) = \frac{\partial Q}{\partial T_s}. \tag{32}$$

Substituting (31) together with (29) into (14), one obtains the equation

$$\frac{z}{Q}\left(R\frac{dT_s}{d\tau} + \frac{\partial Q}{\partial \tau}\right) + \frac{\sqrt{az}}{\lambda} P\left(f_1 + \frac{F}{2}\right) = f_1, \tag{33}$$

which jointly with (28) enables one to find the function $f_1(z)$ for any time instant provided the function $T_s(\tau)$ is known.

It follows from Table 1 that the curve $F(f_1)$ can be approximated on the interval $-2.3 \le f_1 \le 0$ by the straight line

$$F = c - bf_1. \tag{34}$$

By using the method of least squares, one finds the values c = 1.247 and b = 1.460.

The latter enables one to find f_1 in (33) in an explicit form:

$$f_1 = \left[\frac{z}{Q}\left(R\frac{dT_s}{d\tau} + \frac{\partial Q}{\partial \tau}\right) + \frac{cP}{2\lambda}\sqrt{az}\right] \Big/ \left[1 - \frac{1-0.5b}{\lambda} P\sqrt{az}\right]. \tag{35}$$

By now eliminating f_1 from (30), with the aid of (34) and (35) one obtains an equation for z:

$$\frac{dz}{d\tau} = c - b\left[\frac{z}{Q}\left(R\frac{dT_s}{d\tau} + \frac{\partial Q}{\partial \tau}\right) + \frac{cP}{2\lambda}\sqrt{az}\right] \Big/ \left[1 - \frac{1-0.5b}{\lambda} P\sqrt{az}\right]. \tag{36}$$

The obtained equation has to be solved jointly with (28), since Q, P, and R depend on the quantity T_p, which is not known in advance. In the general case, T_p in (28) cannot be given in terms of z explicitly; it is, therefore, advisable to go over to the variable T_p in (36). The expression

$$z = \lambda^2 (T_p - T_\infty)^2 / aQ^2 \tag{37}$$

obtaintable from (28) is now differentiated with respect to time; this yields

$$\frac{dz}{d\tau} = \frac{\partial z}{\partial T_p}\frac{dT_p}{d\tau} + \frac{\partial z}{\partial T_s}\frac{dT_p}{d\tau} + \frac{\partial z}{\partial \tau} = \frac{2(T_p - T_\infty)\lambda^2}{aQ}\left[\left(1 - P\frac{T_p - T_\infty}{Q}\right)\frac{dT_p}{d\tau}\right.$$

$$\left. - \frac{T_p - T_\infty}{Q} R\frac{dT_s}{d\tau} - \frac{T_p - T_\infty}{Q}\frac{\partial Q}{\partial \tau}\right]. \tag{38}$$

By inserting (37) and (38) into (36), an equation is obtained with the single unknown T_p:

$$\frac{2\lambda^2 (T_p - T_\infty)}{aQ^2}\left(1 - P\frac{T_p - T_\infty}{Q}\right)\frac{dT_p}{d\tau} = \tag{39}$$

TABLE 2. Comparison of the Results in Evaluating Θ for Heat Transfer by Newton's Law (m = 1) According to [2] (upper figure) and According to the Parametric Method (lower figure)

$\frac{1}{2\sqrt{Fo_x}}$	$Bi_x\sqrt{Fo_x}$: 0,01	0,1	1	10	50
0	0,0113 / 0,0111	0,1035 / 0,1023	0,5725 / 0,5780	0,9439 / 0,9508	0,9887 / 0,9905
0,2	0,0077 / 0,0076	0,0715 / 0,0711	0,4136 / 0,4232	0,7244 / 0,7586	0,7665 / 0,8045
0,4	0,0050 / 0,0050	0,0469 / 0,0466	0,2830 / 0,2935	0,5256 / 0,5834	0,5621 / 0,6320
0,8	0,0018 / 0,0018	0,0172 / 0,0169	0,1111 / 0,1232	0,2305 / 0,3102	0,2520 / 0,3543
1,2	0,0006 / 0,0005	0,0050 / 0,0050	0,0328 / 0,0433	0,0778 / 0,1446	0,0871 / 0,1763

$$= c\left[1 - \frac{b}{2}\,\frac{P(T_p - T_\infty)}{Q - (1-0.5b)P(T_p - T_\infty)}\right] + \frac{2\lambda^2 (T_p - T_\infty)^2}{aQ^3}$$

$$\times\left(R\frac{dT_s}{d\tau} + \frac{\partial Q}{\partial \tau}\right)\frac{Q(1-b) - (1-0.5b)P(T_p - T_\infty)}{Q - (1-0.5b)P(T_p - T_\infty)}. \quad (39)$$

As a result of these transformations, the solution of the problem (1)-(3), irrespective of the function $Q = Q(T_p, T_s, \tau)$, reduces to integrating the differential equation (39) of the first order together with the initial condition $T_p = T_{p0} = T(0, 0)$ and with the subsequent evaluation of the value of z and f_1 by using the formulas (37) and (35), respectively.

In the case of T_s = const and Q not depending on time explicitly, the variables in Eq. (39) can be separated and the solution can be written as

$$\frac{a\tau}{\lambda^2} = \int_{T_{p0}}^{T_p} \Lambda(T_p)\,dT_p, \quad (40)$$

where

$$\Lambda(T_p) = \left[2\frac{T_p - T_\infty}{Q^2}\left(1 - P\frac{T_p - T_\infty}{Q}\right)\right] \Big/ c\left[1 - \frac{b}{2}\,\frac{P(T_p - T_\infty)}{Q - (1-0.5b)P(T_p - T_\infty)}\right]. \quad (41)$$

To be able to determine the error of the proposed approach and to describe the procedure for evaluating the temperature field, we analyze in detail the solution of the problem (1)-(3) in the case in which the function Q in the boundary conditions (2) is given by

$$Q(T_p) = \alpha_m (T_s^m - T_p^m),\ \alpha_m = \text{const},\ T_s = \text{const}.$$

The expression (41) is then given by

$$\Lambda(T_p) = \frac{2}{c\alpha_m^2}\,\frac{(T_p - T_\infty)[T_s^m - T_p^m + (1-0.5b)\,mT_p^{m-1}(T_p - T_\infty)]}{(T_s^m - T_p^m)^3}. \quad (42)$$

In the particular case of m = 1 (Newton's law of heat transfer — a linear problem) or m = 4 one has T_s = 0 (radiative transfer heat by in vacuum) with the initial condition m = 4, $T_0(x) = T_\infty$, one obtains from (40) and (42)

$$\frac{\alpha_1^2 a\tau}{\lambda^2} = \frac{2}{c}\int_0^{\Theta_p} \frac{\Theta_p(1 - 0.5b\Theta_p)}{(1-\Theta_p)^3}\,d\Theta_p \ \text{for}\ m = 1, \quad (43)$$

$$\frac{\alpha_4^2 T_\infty^6 a\tau}{\lambda^2} = \frac{2}{c}\int_0^{\Theta_p} \frac{\Theta_p[1 + \Theta_p(3-2b)]}{(1-\Theta_p)^9}\,d\Theta_p \ \text{for}\ m = 4. \quad (44)$$

TABLE 3. Comparison of the Results in Evaluating Θ_p for Radiative Heat Transfer (m = 4) for $T_s = 0$ According to [5] (upper figure) and According to the Parametric Method (lower figure)

$1/2\sqrt{Fo_x}$	$(Ki_x)_0\sqrt{Fo_x}$				
	0,01	0,1	1	10	100
0	$\frac{0{,}01}{0{,}0108}$	$\frac{0{,}09}{0{,}0849}$	$\frac{0{,}31}{0{,}3184}$	$\frac{0{,}57}{0{,}5746}$	$\frac{0{,}76}{0{,}7502}$

In the above $\Theta_p = (T_p - T_\infty)/(T_s - T_\infty)$ is the dimensionless temperature of the surface.

It can easily be observed that the solutions (43) and (44) admit the traditional criteria formulation, namely,

$$Bi_x^2 Fo_x = \Omega_1(\Theta_p),\ (Ki_x)_0^2\, Fo_x = \Omega_4(\Theta_p),$$

where $\Omega_1(\Theta_p)$ and $\Omega_4(\Theta_p)$ are the right-hand sides of the relations (43) and (44) and can be tabulated. A computation formula for the sought temperature field can be obtained from (27) having substituted (37) in it:

$$\Theta = \Phi\Theta_p,$$

where $\Theta = (T - T_\infty)/(T_s - T_\infty)$ is the dimensionless temperature.

The function $\Phi(\eta, f_1)$ is found by using Table 1; to apply it, one evaluates in advance the quantity f_1 by using the formula

$$f_1 = \begin{cases} \dfrac{c\Theta_p}{b\Theta_p - 2} & \text{for}\quad m = 1, \\ \dfrac{2c\Theta_p}{\Theta_p(2b-3)-1} & \text{for}\quad m = 4, \end{cases}$$

which follow from (35) and the quantity

$$\eta = \begin{cases} \dfrac{Bi_x}{Bi_{\delta^*}} & \text{for}\quad m = 1, \\ \dfrac{(Ki_x)_0}{(Ki_{\delta^*})_0} & \text{for}\quad m = 4, \end{cases}$$

where

$$Bi_{\delta^*} = -\frac{2f_1}{(2-b)f_1 + c};\ (Ki_{\delta^*})_0 = -\frac{f_1[(3-2b)f_1 + 2c]^3}{16[(2-b)f_1 + c]}. \qquad (45)$$

The expressions (45) were obtained from (37).

Similarly as in [3], the calculation results were processed in the coordinates $\Theta_1 = \Theta_1(Bi_x\sqrt{Fo_x})$ and $\Theta_4 = \Theta_4[(Ki_x)_0\sqrt{Fo_x}]$ for $1/2\sqrt{Fo_x}$ = const and they are shown in Tables 2 and 3 and in Fig. 1. The exact solution of the linear problem given in [3] enables one to estimate the error of the parametric method. It follows from Table 2 that this method produces almost exact results for the surface temperature. If one allows an error of ±1% from the maximal temperature, then one can see that this requirement is satisfied in a sufficiently large variation domain of the generalized argument $0 \le Bi_x \le \sqrt{Fo_x} \le 1$. In the remaining range of the variation of arguments the error never exceeds 10%. Bearing in mind that in practice one comparatively rarely comes across the values $10 \le Bi_x\sqrt{Fo_x} \le 50$, the accuracy of the method should be considered as good.

For the case of m = 4 no exact solutions were found as stated in [1]. In [5], the curve of the temperature of the body surface versus time obtained by numerical integration of the heat-conduction equation is shown. In Table 3 the values of the surface temperature obtained with the aid of the parametric method were compared with the values obtained from a graph shown in [5]. It follows from Table 3 that the accuracy of the proposed method is high. For practical calculations the graphs of the relations $\Theta_4 = \Theta_4[(Ki_x)_0\sqrt{Fo_x}]$ for $1/2\sqrt{Fo_x}$ = const, which enable one to calculate the temperature field of a semiinfinite body, are shown.

The problems solved above seem to be the simplest in their own class. More complex problems, which are important in practice [such as heat transfer by free convection (m = 5/4), radiative heat transfer with $T_s \ne 0$, radiative–convective heat transfer, etc.], can also be solved in a similar manner.

In conclusion, one should observe that this method can, in principle, be extended to bodies of finite dimensions, and (which is especially important) its use enables one to solve conjugate heat-transfer problems relatively easily.

NOTATION

T is the temperature, °K;
Θ is the dimensionless temperature;
q is the heat-flux density, W/m^2;
Q is the heat-flux density on body surface, W/m^2;
φ is the dimensionless density of heat flux;
x is the coordinate, m;
η is the dimensionless coordinate;
δ^* is the thickness of filling, m;
τ is the time, sec;
f_k is the dimensionless similarity parameter;
a is the coefficient of thermal diffusivity of body; m^2/sec;
λ is the coefficient of thermal conductivity of body; $W/m \cdot °K$;
α_m is the proportionality coefficient with dimension dependent on m;
m is the coefficient describing the heat-transfer law;
$Bi_x = \alpha_1 \varkappa/\lambda$ is the Biot number;
$(K_{1x})_0 = Q_0 x/\lambda(T_s - T_{p0})$ is the Kirpichev criterion;
$Fo_x = a\tau/x^2$ is the Fourier number.

Indices

0 is the initial value;
p is the body surface;
∞ is the point at infinity;
s is the surrounding medium.

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